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BIFURCATION OF A TRANSONIC IDEAL-GAS FLOW AROUND A SYMMETRIC AIRFOIL WITH A CONCAVITY

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A transonic flow around a symmetric airfoil with a concavity in its central part has been numerically investigated. The dependences of the lift coefficient on the Mach number of the incident flow M_∞ and on the angle of attack α were determined. It is shown that, depending on M_∞, in the flow past the upper part of the indicated airfoil there can arise one or two supersonic-flow regions. It has been established that, at fairly large angles of attack, the coalescence and separation of supersonic-flow regions are realized in a discrete way. For these angles of attack, singular Mach numbers M_s, in the neighborhood of which the structure of the flow is transformed, were determined and the physical processes occurring in this case were analyzed. It was found that the flow being considered is characterized by a large hysteresis in M_∞.

Introduction. In a high-velocity subsonic flow around an airfoil there can arise a supersonic-flow region (supersonic region) in the near-airfoil zone. If an airfoil has a small concavity in its central part, several (two or more) supersonic regions can arise at its lower and upper sides. It has been established [1] that a change in the Mach number of the incident flow M_∞ can cause a restructuring of the pattern of the flow near the airfoil. An increase in M_∞ causes the coalescence of supersonic regions into one region, and a supersonic region separates into subregions of smaller dimensions when M_∞ decreases.

In [1–3], a transonic flow in a channel with a bump, simulating an airfoil, was numerically investigated. It has been established in these works that a steady flow in this channel changes in a discrete way when supersonic regions coalesce or separate and that there exist singular (in the terminology of [1–3]) Mach numbers $M_\infty = M_s$, in the neighborhood of which the indicated flow experiences structure transformations, i.e., becomes structurally unstable; this phenomenon was called the structural instability. The data obtained in [1–3] aid in understanding the reasons for the nonuniqueness of transonic flows, noted earlier by other authors [4, 5].

Airfoils with a concave central part have attracted considerable attention of researchers [6–12] because they make it possible to attenuate shock waves and their shape can be adapted to the regime of transonic flows around them. For the purpose of control of flows around these airfoils, it was proposed in [7, 8] to make small-length smooth bumps near their tail. Then, in [9–12], data on the stability of a flow around such an airfoil were obtained. However, the physical processes occurring in this case remain unclear. For example, the conclusions made in [10, 12] are somewhat contradictory.

In the present work, we numerically investigated a transonic inviscid flow around a symmetric airfoil with a concavity in its central part:

$$y(x) = \pm [0.06\sqrt{1-(2x-1)^2}\,(1-x^{16})^2 - 3x^3\,(1-x)^5] \; . \tag{1}$$

The calculations were performed for an incident flow with a Mach number M_∞ of 0.8–0.89. These calculations have shown that in a flow around an airfoil defined by expression (1) there can arise one or two supersonic regions in the upper part of the airfoil and that an increase in the Mach number of the incident flow leads to a bifurcation of the flow near the airfoil. It has been established that, when M_∞ changes, the structure of this flow is transformed continuously at small angles of attack ($\alpha < 0.8$) and in a discrete way at large angles of attack. The regimes of flow around the airfoil being considered at α = 1.4 and 1.1 were investigated in detail. For these angles of

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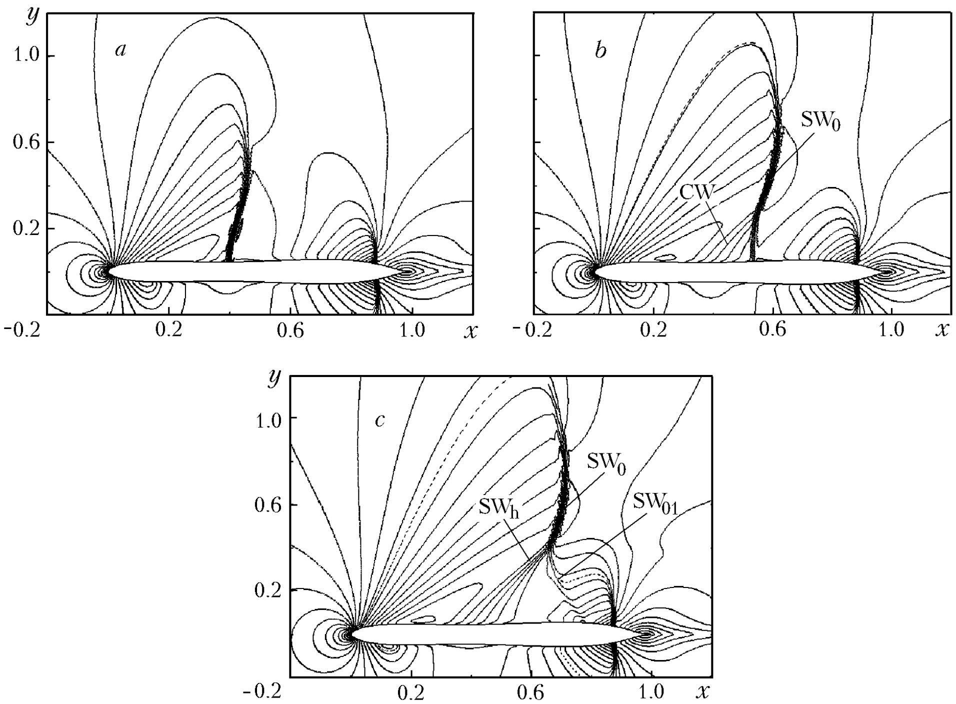

Fig. 1. Isolines of the Mach number M (α = 1.4): M_∞ = 0.800 (a), 0.807 (b), 0.8073 (c).

attack, Mach numbers of the incident flow M_∞, in the small neighborhood of which the flow experiences structure transformations, were determined. The physical reasons for this phenomenon were analyzed.

Formulation of the Problem. Method of Numerical Investigation. A two-dimensional inviscid air flow (with an adiabatic index of 1.4) around a symmetric concave airfoil (1) is considered. The system of Euler equations is solved using the NSC2KE program (tested many times) based on the finite-volume method of the second order of accuracy [6]. Calculations are performed on a nonuniform computational C-shaped grid consisting of triangular elements bunching near the airfoil, in the region of shock waves, and in the wake. The outer boundary of the computational region is at a distance of 15 chord lengths from the airfoil. Along the outer boundary, 543 nodes are located, and 76 nodes are in the wake. In the "radial" directions (close to the direction of the normal to the airfoil and the wake) 159 grid pitches are prescribed. The time integration is performed using the explicit Runge–Kutta algorithm.

The slip condition is set at the surface of the airfoil. At the outer boundary of the airfoil, the angle of attack α and the Mach number $M_\infty < 1$ are prescribed. Stationary solutions are found by the ascertainment method. In this case, the initial conditions are the parameters of a uniform flow or the field of a nonuniform flow, obtained by calculating a flow around the airfoil at other values of M_∞ and α.

Discussion of the Results Obtained. 1. Our calculations have shown that, throughout the range of Mach-numbers being investigated $0.8 \le M_\infty \le 0.89$, at angles of attack α = 1.4 and 1.1 at the lower side of the airfoil two small supersonic regions arise and, at the upper side there can arise a flow with two supersonic regions that coalesce into one region when M_∞ increases. Figure 1 shows the patterns of steady flow at an angle of attack α = 1.4 and an increasing value of the Mach number. At small values of M_∞, two supersonic regions arise at the upper side of the airfoil. This pattern persists as long as M_∞ = 0.807. However, when M_∞ increases further (to 0.8073), the two supersonic regions coalesce rapidly into one region. Thus, the singular Mach numbers fall within the range $0.807 < M_\infty < 0.8073$. At an angle of attack α = 1.1, they take values from the range $0.8165 < M_\infty < 0.817$.

2. The discrete structure transformations of a transonic flow around the airfoil being investigated are usually accompanied by abrupt large changes in the lift coefficient C_L; in this case, C_L can change by several times. This is explained by the large dependence of this parameter on the ratio between the sizes of the supersonic regions at the upper and lower sides of the air foil — C_L can increase sharply when the supersonic regions at the upper side of the

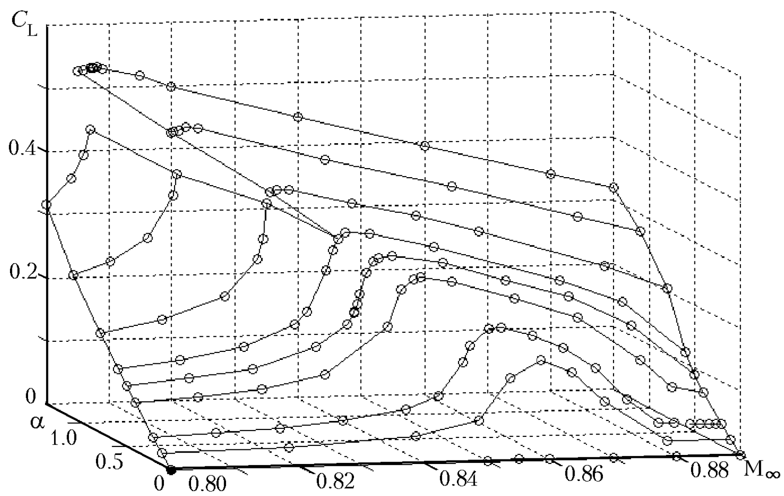

Fig. 2. Dependence of the lift coefficient on the angle of attack and the Mach number.

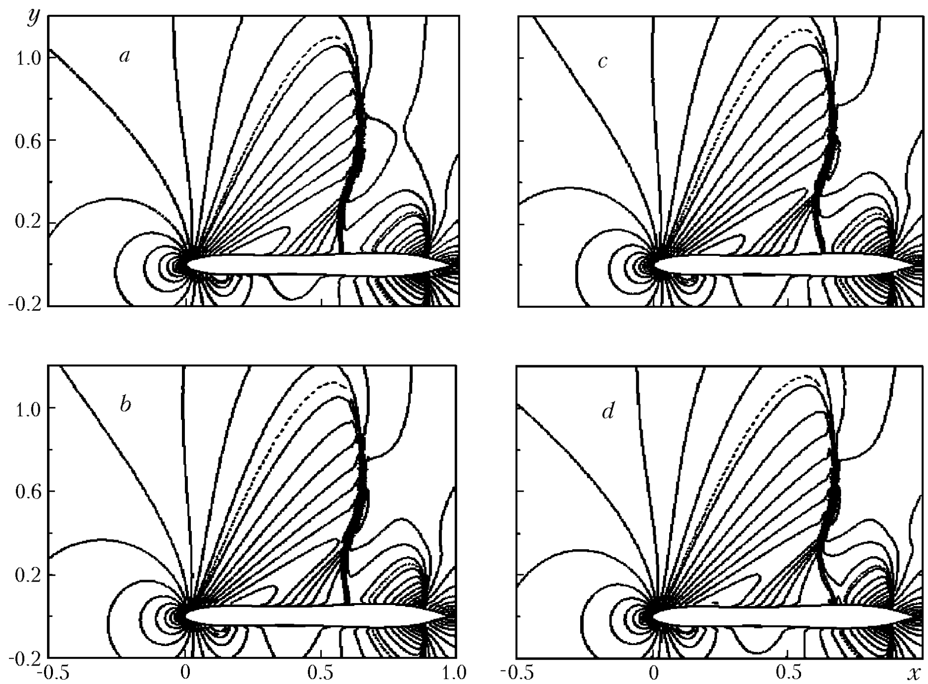

Fig. 3. Isolines of the Mach number M. Bifurcation of the flow in the case of increase in M_∞ from 0.807 to 0.8073, α = 1.4. Shift of the shock wave downstream with change in the inclination of its base (a–c), beginning of the coalescence of supersonic regions (d).

airfoil coalesce and decrease when a single supersonic region separates into subregions of smaller dimensions or the supersonic regions at the lower side of the airfoil coalesce.

Figure 2 shows the dependence of C_L on the angle of attack α and the Mach number M_∞. A peculiarity of the concave air foil being considered is that C_L increases markedly with increase in M_∞ even before the supersonic regions at the upper side of the airfoil begin to coalesce. This is illustrated well by Fig. 2. It is seen from this figure that, at fairly large angles of attack ($\alpha > 0.8$), the bifurcation proceeds in a discrete way and, consequently, the curves α = 1.8, 1.1, and 1.4, forming the surface C_L (α, M_∞) discontinue. Such behavior of C_L is explained by the fact that

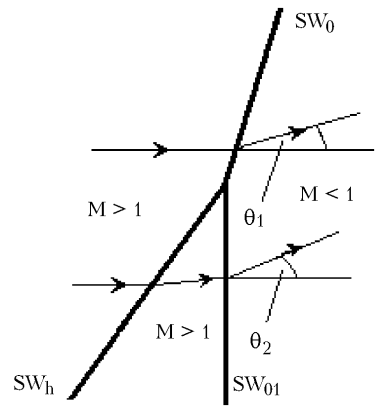

Fig. 4. Scheme of the triple configuration of shock waves.

the dimensions of the first supersonic region at the upper side of the airfoil (located upstream of it) increase rapidly with increase in M_∞ even before the bifurcation point is attained, which is apparent when the flow patterns in Fig. 1a and b are compared. In this case, at the angles of attack being considered, an increase in M_∞ does not cause a marked increase in the dimensions of the supersonic regions at the lower side of the airfoil.

It should be noted that, even though the structure of the flow being considered is transformed in a discrete way at angles of attack $\alpha = 1.4$ and 1.1 (a discontinuity of the surface C_L (α, M_∞), Fig. 2) and C_L changes abruptly at a bifurcation point, the discontinuity of the surface C_L (α, M_∞) is insignificant. This is explained by the fact that restricting this flow does not cause a very large increase in the dimensions of the supersonic region at the upper side of the airfoil (Fig. 1b and c).

3. We now consider the structure of the flow near the upper side of the airfoil and its evolution at $\alpha = 1.4$ in greater detail. At $M_\infty = 0.800$, a shock wave closing the first supersonic region intersects the airfoil surface near the smallest *y* coordinate, i.e., at the "bottom" of the concavity. When M_∞ increases, the first supersonic region increases and the shock wave SW_0, closing this region, shifts downstream (Fig. 1b). As a result of the passage of a supersonic flow around the concave region of the airfoil surface, there arises a compression wave CW (Fig. 1b).

At $M_\infty = 0.8073$, the flow being considered becomes unstable and, after a number of intermediate unstable states (Fig. 3), it takes a triple configuration with shock waves: $SW_h - SW_{01} - SW_0$ (Fig. 1c). The separation of the single shock front closing the first supersonic region and the formation of the λ-like configuration of shock waves can be explained in the following way. When the dimensions of the first supersonic region increase and the shock wave closing this region shifts downstream, the near-wall gas flow in the first supersonic region passes over the surface with a larger curvature because the larger part of the concavity is in the supersonic region. This leads to a gradual enhancement of the compression wave CW. Because of the shift of the shock wave closing the supersonic region downstream, the point at which the first characteristic of the compression wave attains this shock wave moves away from the surface of the airfoil. As a result, there appears a tendency for the formation of a hanging compression shock SW_h (Fig. 1c) characteristic of steady-state regimes with a single supersonic region at the upper side of the airfoil.

The formation of a hanging compression shock near the surface of the airfoil at a small angle of inclination of the shock wave SW_{01}, closing the first supersonic region, would lead to the formation of a triple configuration similar to the configuration shown in Fig. 4. In this case, as simple estimates show, after the passage through the two compression shocks SW_h and SW_{01}, the resulting angle θ_2 of inclination of the velocity vector would be larger than the angle θ_1 of inclination of the velocity vector of the flow passing through the single compression shock SW_0. This configuration is impossible from the theoretical standpoint. It seems likely that it is precisely the coordination of the angles of turn of the flows passing below and above the triple point that leads to a change in the angle of inclination of the shock-wave base. This causes a shift of the shock-wave base downstream and an increase in the angle of its inclination. After the shock-wave base reaches the sonic line restricting the second supersonic region, the supersonic regions coalesce into one zone. Thus, the restructuring of the flow leads to the formation of a triple configuration of shock waves (Fig. 1c).

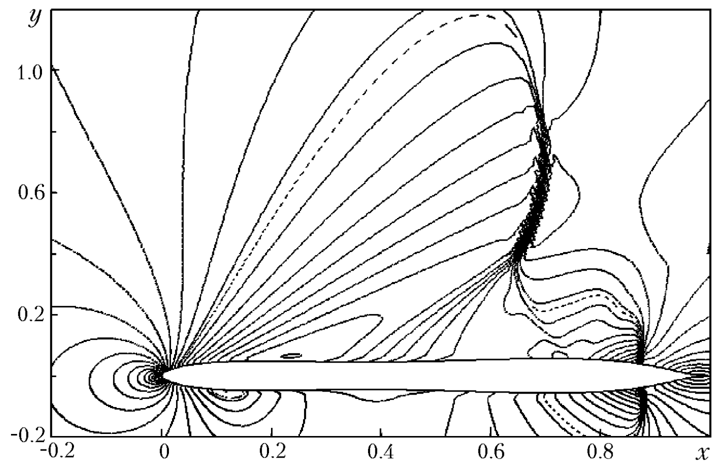

Fig. 5. Isolines of the Mach number M under the conditions where M_∞ = 0.805, α = 1.4, and there exists a hysteresis of flow in M_∞.

4. The investigation performed in the present work has shown that, at angles of attack α = 1.4 and 1.1, the stationary solution of the problem being considered depends on the initial conditions. In the neighborhood of M_s there can arise different steady flows depending on the side (larger or smaller values of M_∞) on which M_s is approaches. Namely, it has been established for α = 1.4 that the scheme of flow with a single supersonic region at the upper side of the airfoil persists for a certain time when the Mach number decreases from M_∞ = 0.808. Figure 5 shows the pattern of a steady flow realized when M_∞ = 0.805. When this pattern is compared to the pattern obtained for M_∞ increasing to 0.8073 (Fig. 1c), it is apparent that, in the case where M_∞ decrease, the depth of the deflection of the sonic line increases gradually. Moreover, on the sonic line there appears one more deflection that is due to the reflected shock wave. This scheme of flow persists as long as M_∞ = 0.8046, and at M_∞ = 0.804 the supersonic region separates into two subregions. Therefore, the hysteresis in M_∞ is equal to approximately 0.0027. The calculations for α = 1.1 have shown that the analogous process takes place in the case where M_∞ decreases from 0.8157 to 0.8154, i.e., the hysteresis is small in this case. Thus, a flow around a concave airfoil with a fairly large angles of attack, at which the structure of the flow transforms in a discrete way, is characterized by a marked hysteresis in M_∞, the width of which increases with increase in the angle of attack.

Conclusions. It is shown that, in the case where an inviscid fluid flows around a concave airfoil, the pattern of the flow near the airfoil can change in a discrete way when the Mach number of the incident flow changes continuously. A bifurcation of the flow is accompanied by an abrupt change in the lift coefficient. At angles of attack α = 1.4 and 1.1 there exist singular Mach numbers M_s, in the neighborhood of which the structure of the flow is transformed. The values of these numbers were determined and the physical reasons for the bifurcation of the flow with increase in M_∞ were explained.

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NOTATION

C_L, lift coefficient; M_∞, Mach number of the incident flow; M_s, singular Mach number; x, y, Cartesian coordinates; α, angle of attack, deg; θ_1, θ_2, direction of the velocity vector, deg; SW, shock wave; CW, compression wave. Subscripts: L, lift coefficient; ∞, point at infinity; s, singular; h, hanging.

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